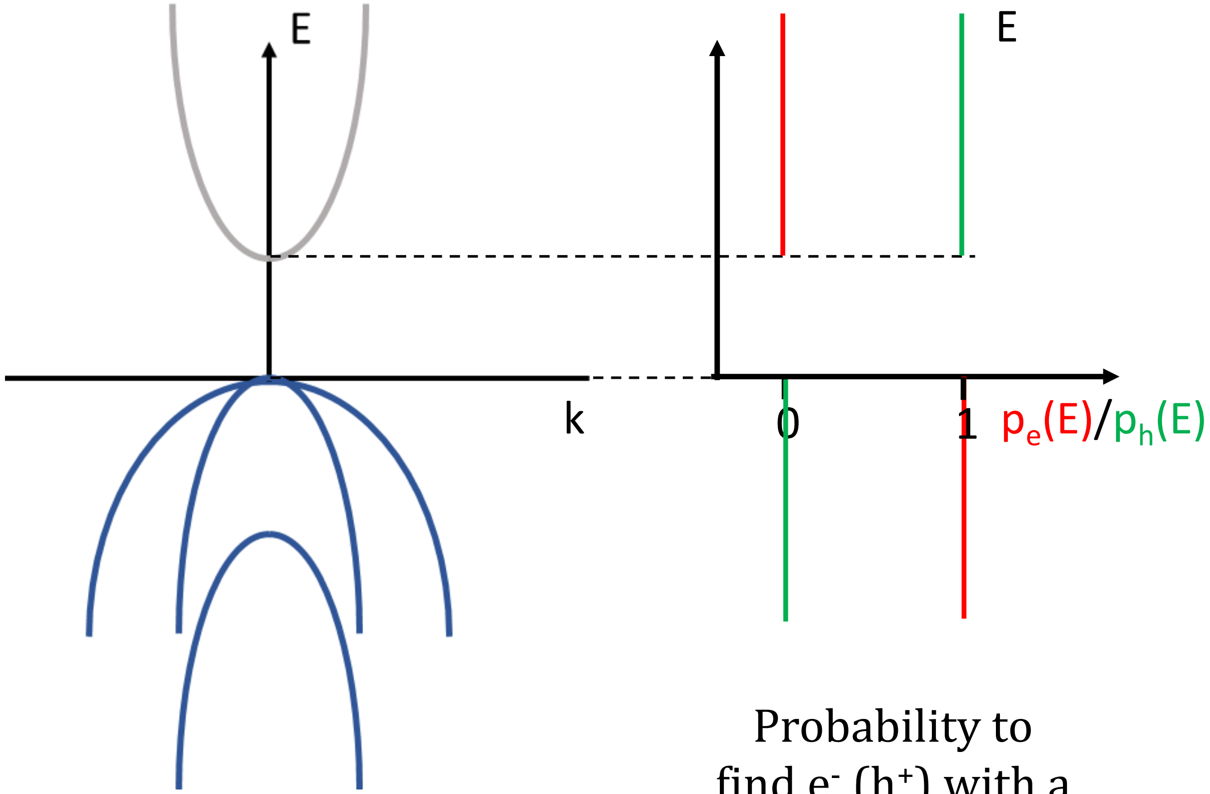
Class 04
Charge density in intrinsic semiconductors

25.02.2025

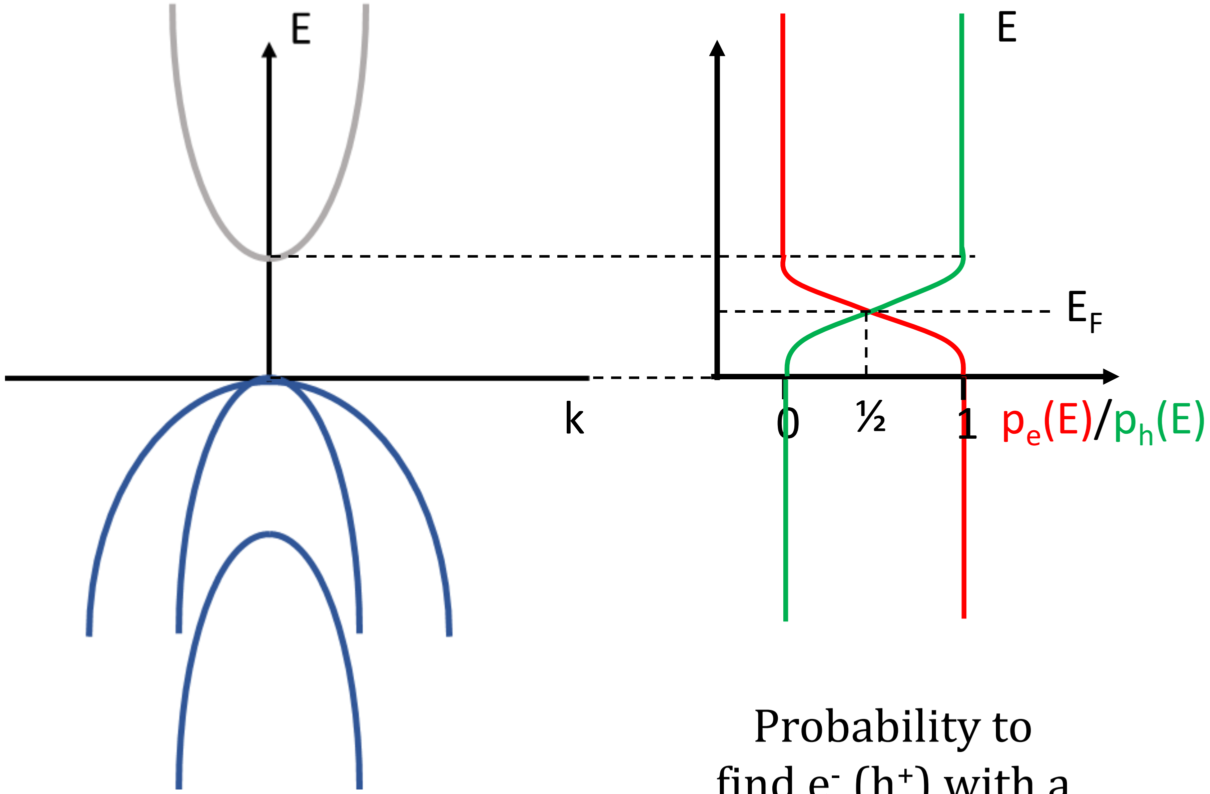
- ❑ Fermi-Dirac distribution and Fermi energy
- ❑ Carrier Statistics
- ❑ Charge density engineering
  - Effect of band gap
  - Effect of dimensionality
  - Effect of temperature
  - Effect of electric field
  - Effect of confinement

Carrier Statistics



Probability to find e^- (h^+) with a certain energy at T = 0K

Carrier Statistics



Probability to find e⁻ (h⁺) with a certain energy at T = 0K

Fermi-Dirac distribution

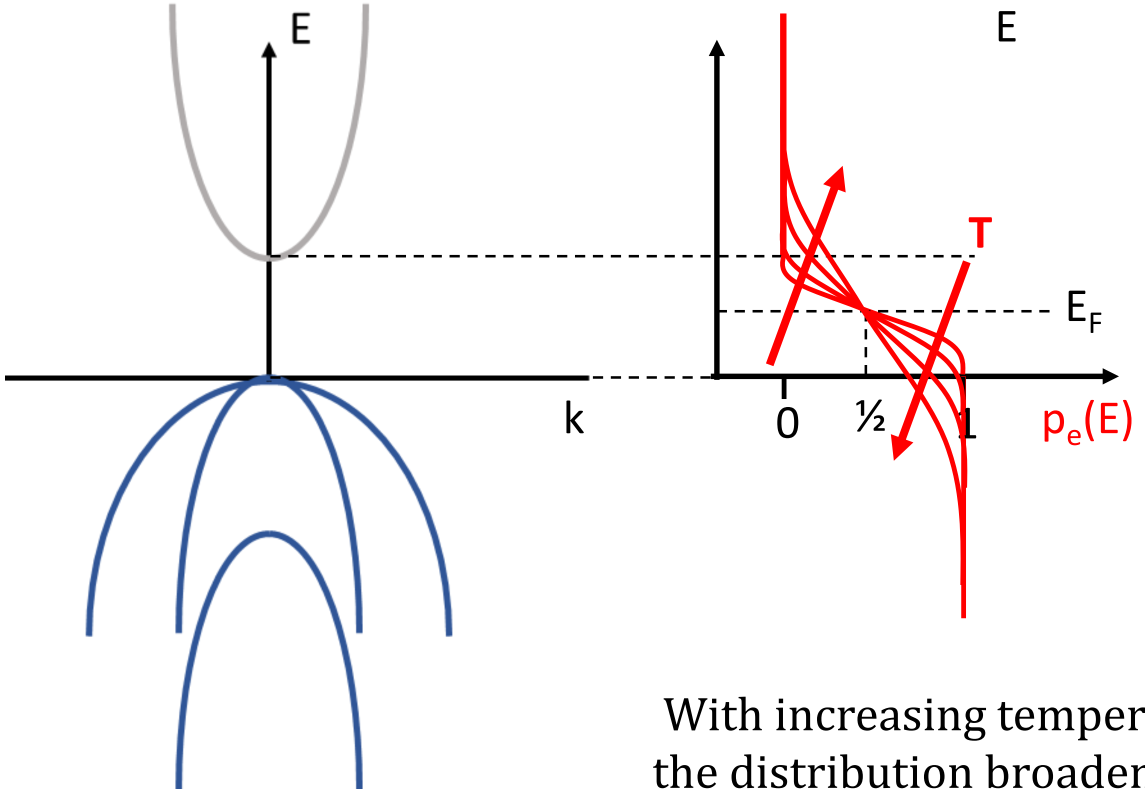
$$f_e(E) = \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)+1}\,.$$

$$f_h(E) = 1 - \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)+1} = \frac{1}{\exp\left(-\frac{E-E_F}{kT}\right)+1}$$

Can you give the definition of E_F starting from the general expression of FD distribution?

Please note that by definition, E_F represents the chemical potential of the crystal.

Carrier Statistics



With increasing temperature the distribution broadens and the probability to find e^-(h^+) in the conduction (valence) band is higher than 0

Fermi-Dirac distribution

$$f_e(E) = \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)+1} .$$

$$f_h(E) = 1 - \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)+1} = \frac{1}{\exp\left(-\frac{E-E_F}{kT}\right)+1}$$

where:

$$f_e(E_F) = fh(E_F) = \frac{1}{2}$$

E_F is the Fermi energy

Please note that by definition, E_F represents the chemical potential of the crystal.

Carrier density

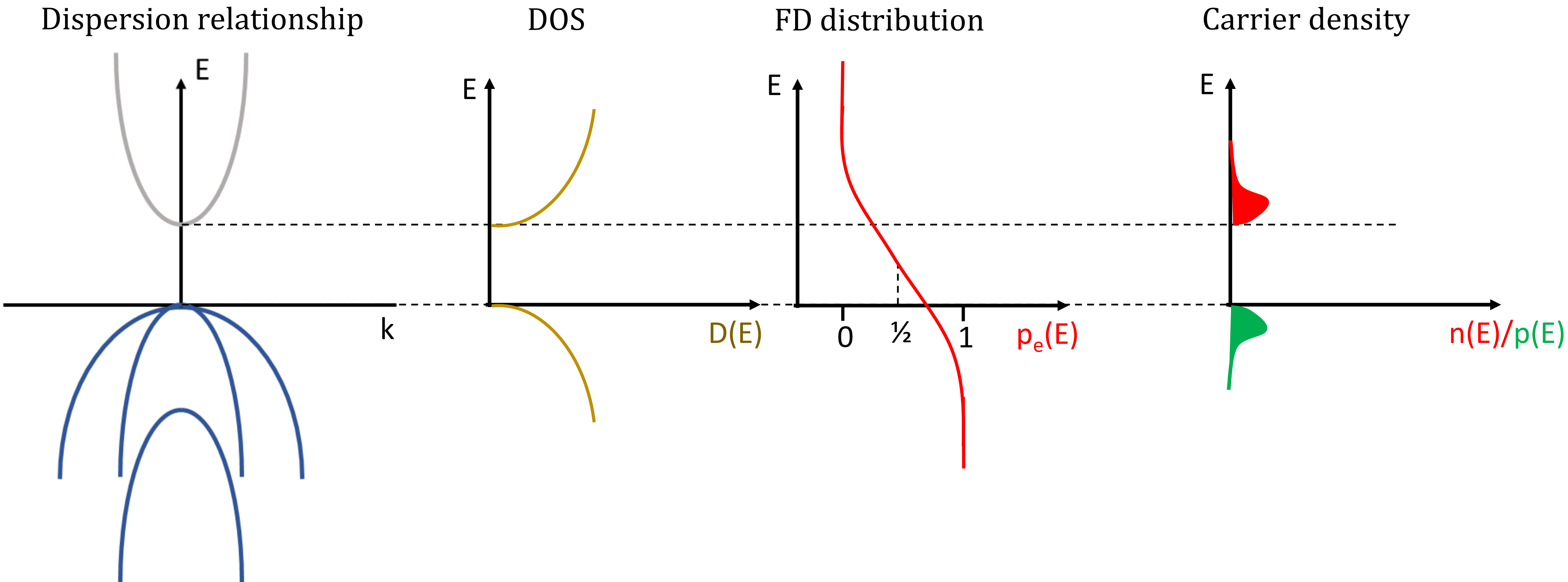
$$n = \int_{E_C}^{\infty} D_e(E)\, f_e(E)\, \mathrm{d}E$$

Conduction band — Density of states — Fermi-Dirac distribution for electrons

and

$$p = \int_{-\infty}^{E_V} D_h(E)\, f_h(E)\, \mathrm{d}E$$

Valence band — Density of states — Fermi-Dirac distribution for holes



Analytical solution to the charge density integral

$$n = \int_{E_C}^{\infty} D_e(E)\, f_e(E)\, dE$$

where:

$$D_e^{3D}(E) = \frac{1}{2\pi^2}\left(\frac{2\, m_{d,e}}{\hbar^2}\right)^{3/2}\sqrt{E - E_C}\,,\ E > E_C$$

$$D_h^{3D}(E) = \frac{1}{2\pi^2}\left(\frac{2\, m_{d,h}}{\hbar^2}\right)^{3/2}\sqrt{E_V - E}\,,\ E < E_V\ .$$

$$f_e(E) = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1}\ .$$

Boltzmann approximation

If $E - E_F \gg kT$,

$$f_e(E) \cong \exp\left(-\frac{E - E_F}{kT}\right)\ .$$

$$n = 2\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2}\exp\left(\frac{E_F - E_C}{kT}\right) = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

$$p = 2\left(\frac{m_h kT}{2\pi\hbar^2}\right)^{3/2}\exp\left(-\frac{E_F - E_V}{kT}\right) = N_V \exp\left(-\frac{E_F - E_V}{kT}\right)$$

where: $N_C = 2\left(\frac{m_e\, kT}{2\pi\, \hbar^2}\right)^{3/2}$ $\quad N_V = 2\left(\frac{m_h\, kT}{2\pi\, \hbar^2}\right)^{3/2}$

N_C (N_V) is called conduction (valence) band edge density of states

E_F for intrinsic semiconductors

$$n = 2\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{E_F - E_C}{kT}\right) = N_C \exp\left(\frac{E_F - E_C}{kT}\right) .$$

$$p = 2\left(\frac{m_h kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{E_F - E_V}{kT}\right) = N_V \exp\left(-\frac{E_F - E_V}{kT}\right) .$$

where: $N_C = 2\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2}$ $\quad N_V = 2\left(\frac{m_h kT}{2\pi\hbar^2}\right)^{3/2}$,

+

ELECTRONEUTRALITY CONDITION

n=p for intrinsic semiconductors

$$N_C\, exp\left(\frac{E_F - E_C}{kT}\right) = N_V\, exp\left(\frac{E_V - E_F}{kT}\right)$$

E_i is the midgap energy

$$\frac{E_F - E_C - E_V + E_F}{kT} = ln\frac{N_V}{N_C}$$

$$\frac{2E_F - 2E_i}{kT} = -\frac{3}{2} ln\frac{m_e^*}{m_h^*}$$

$$E_F = E_i - \frac{3}{4}kT * ln\frac{m_e^*}{m_h^*}$$

Mass action law

ELECTRONEUTRALITY CONDITION

$-n + p = 0$ (for intrinsic semiconductors)

$n = p$

$n * p = n_i^2$ for all the non-degenerate semiconductors

$$n_i = p_i = \sqrt{N_V N_C} \exp\left(-\frac{E_g}{2kT}\right)$$

Intrinsic electron(hole) density

n_i depends on:

- The temperature
- The bandgap
- The (conduction and valence band) DOS

n_i does not depend on:

- Fermi energy

And it is therefore valid also for doped semiconductors

Recap equations

$$n = \int_{E_C}^{\infty} D_e(E)\, f_e(E)\, dE$$

$$n = 2\left(\frac{m_e kT}{2\pi \hbar^2}\right)^{3/2} \exp\left(\frac{E_F - E_C}{kT}\right) = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

$$N_C = 2\left(\frac{m_e kT}{2\pi \hbar^2}\right)^{3/2}$$

$$E_F = E_i - \frac{3}{4} kT * ln \frac{m_e^*}{m_h^*}$$

$$n_i = p_i = \sqrt{N_V N_C} \exp\left(-\frac{E_g}{2kT}\right)$$

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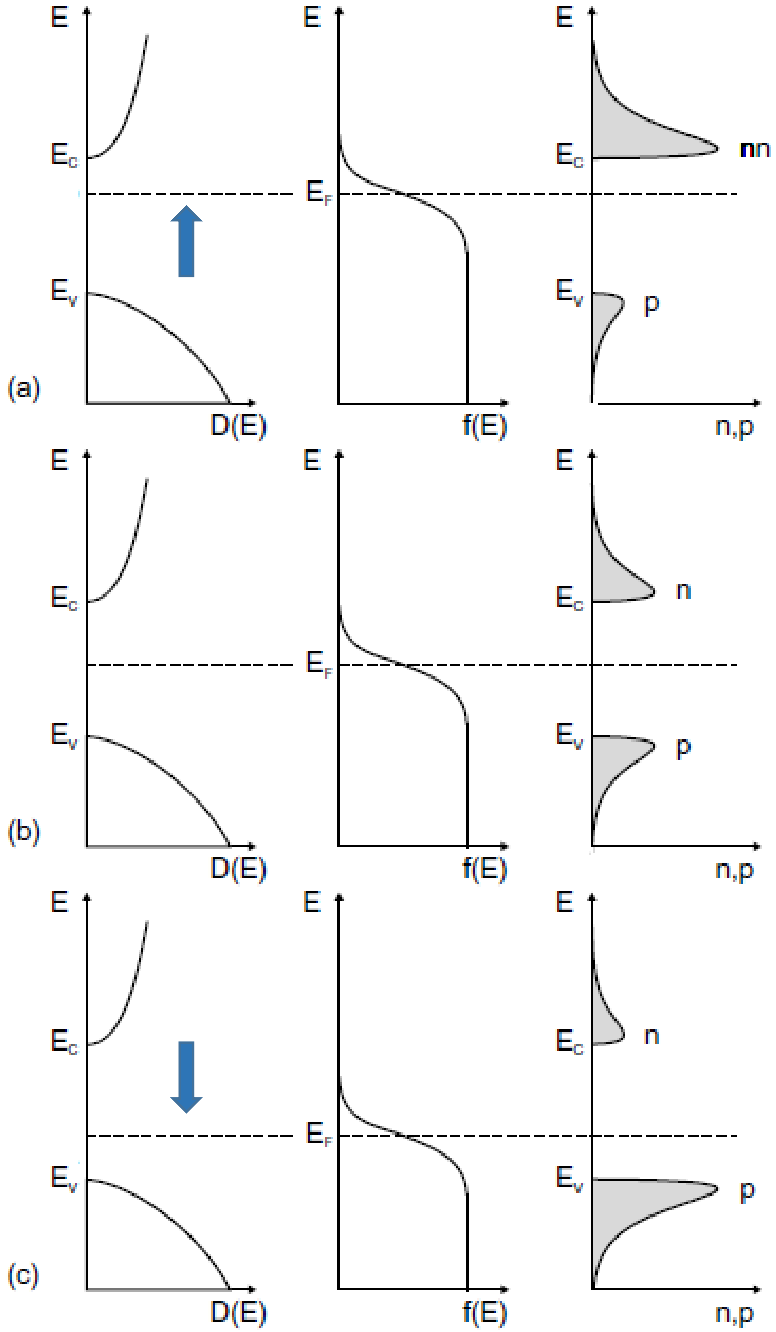
	E_g (eV)	n_i (cm^{-3})
InSb	0.18	1.6×10^{16}
InAs	0.36	8.6×10^{14}
Ge	0.67	2.4×10^{13}
Si	1.124	1.0×10^{10}
GaAs	1.43	1.8×10^{6}
GaP	2.26	2.7×10^{0}
GaN	3.3	$\ll 1$

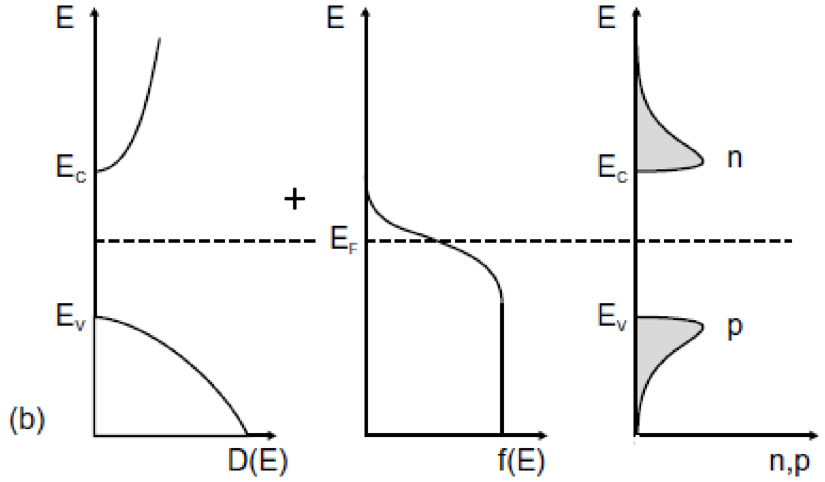
$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

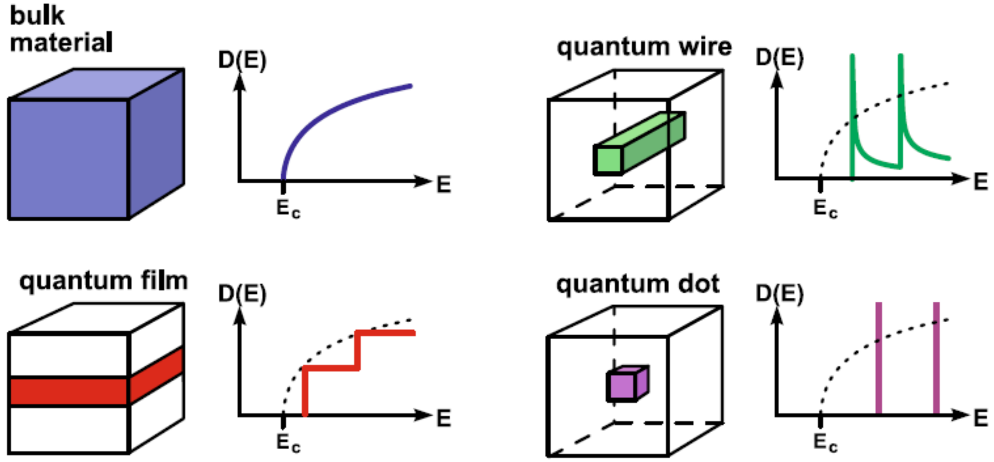


Impact of **dimensionality** on the carrier density

$$n = \int_{E_C}^{\infty} D_e(E)\, f_e(E)\, dE\,,$$

$$p = \int_{-\infty}^{E_V} D_h(E)\, f_h(E)\, dE\,.$$





In bulk

$$N_C = 2\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2}$$

$$N_V = 2\left(\frac{m_h kT}{2\pi\hbar^2}\right)^{3/2}$$

In quantum wire

?

Grundmann Chapter 14

Homework:
Is Nc(Nv) higher in bulk or in nanowire?

Impact of **temperature** on the carrier density

$$n_i = p_i = \sqrt{N_V N_C} \exp\left(-\frac{E_g}{2kT}\right)$$

The bandgap also varies with T due to a different electron-phonon interaction and the variation of the interatomic distance.

How does the band gap change with increasing T?

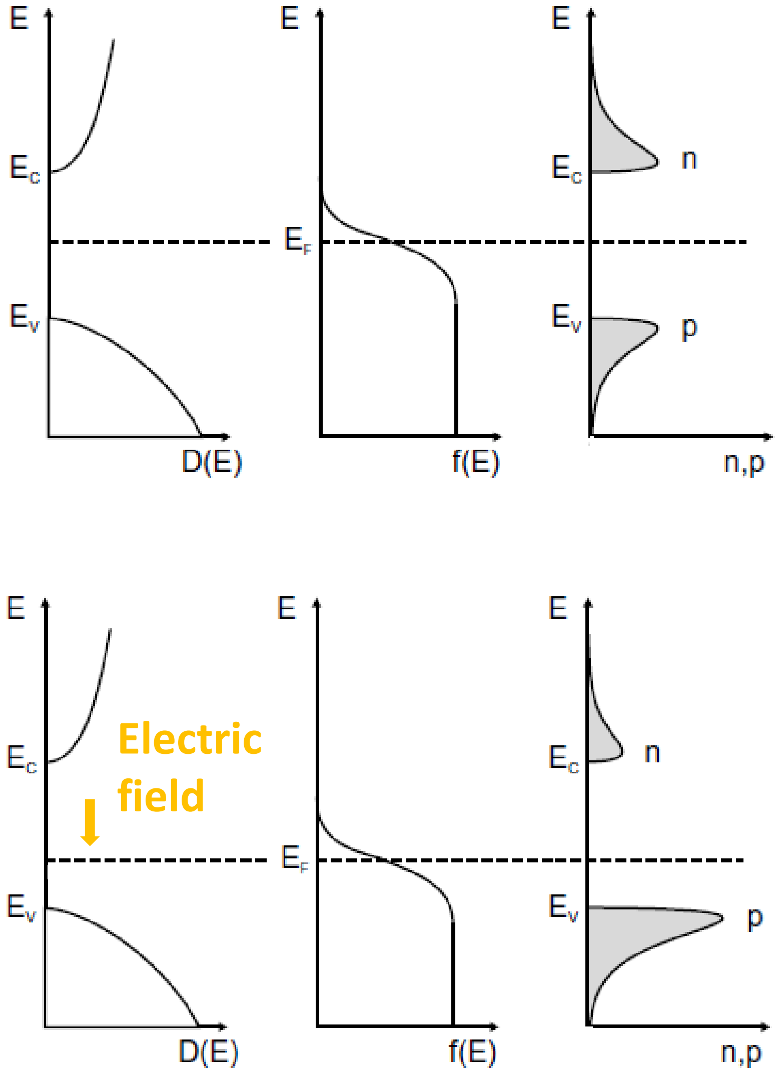
- Larger BG
- Same BG (negligible effect)
- Smaller BG

Which factor is dominant for the n_i?

To be solved in class

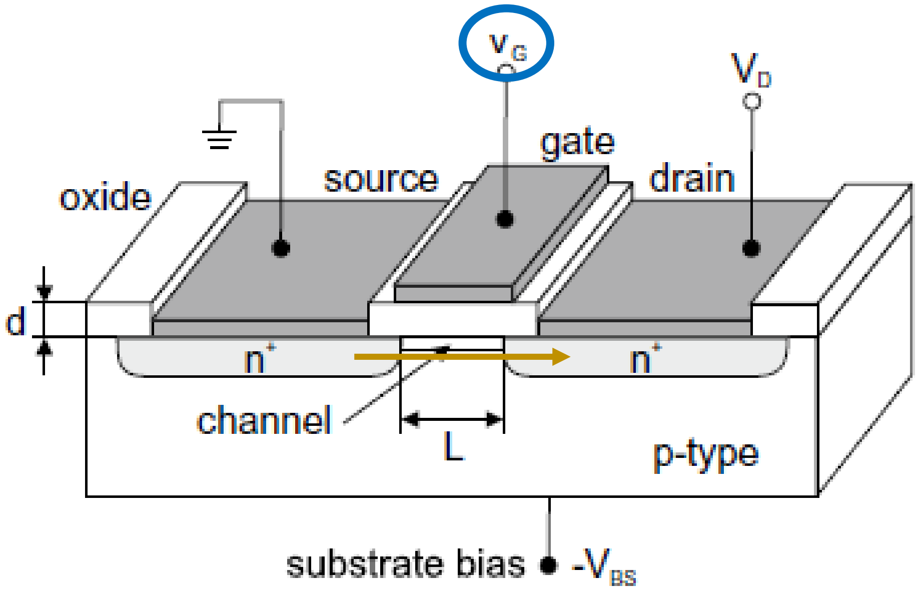
5 minutes

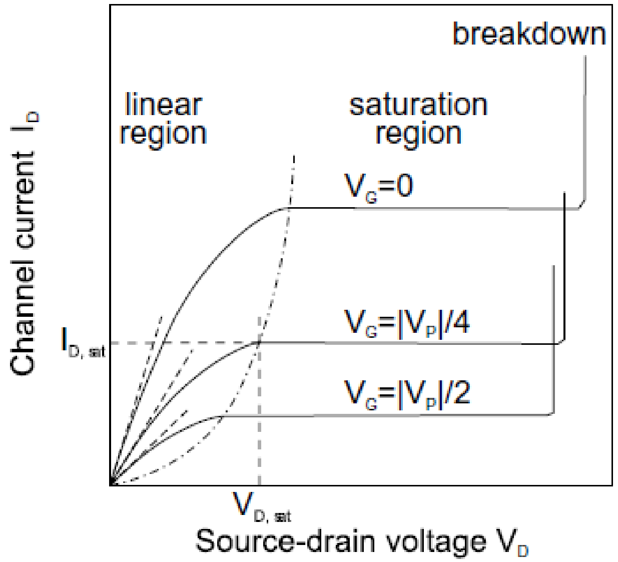
Impact of **electric field** on the carrier density



Electronic switch: Field effect transistor (FET)

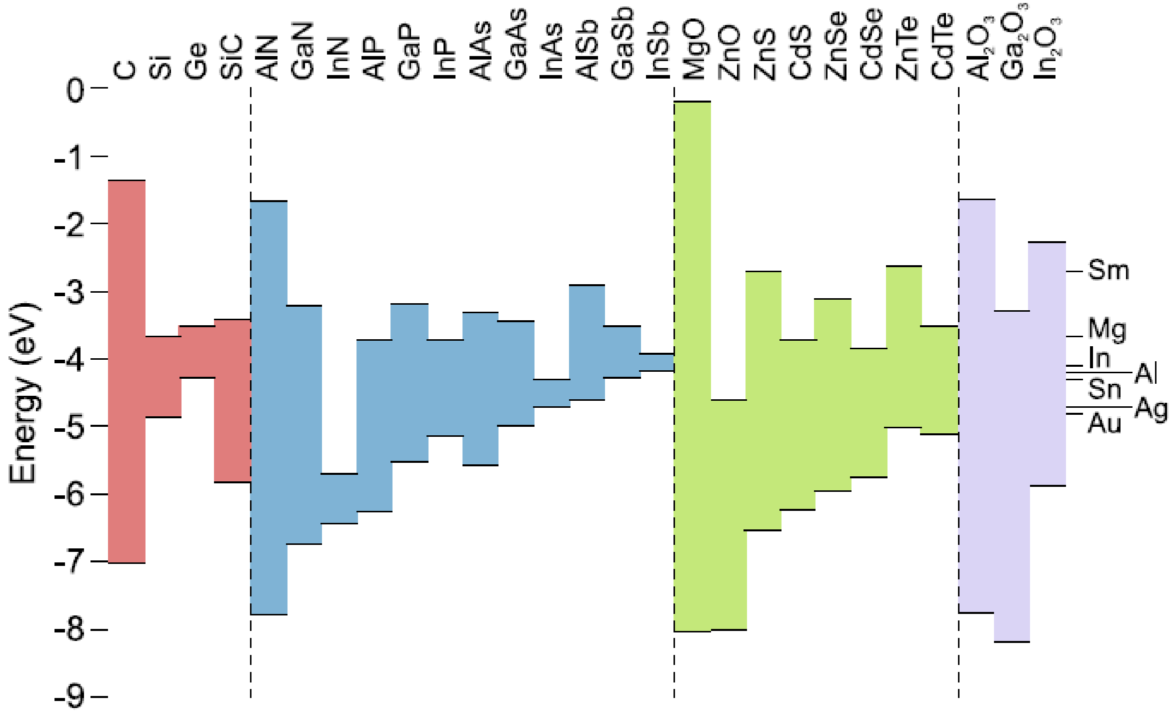
Gate voltage → Controlling the Fermi energy into the «channel»

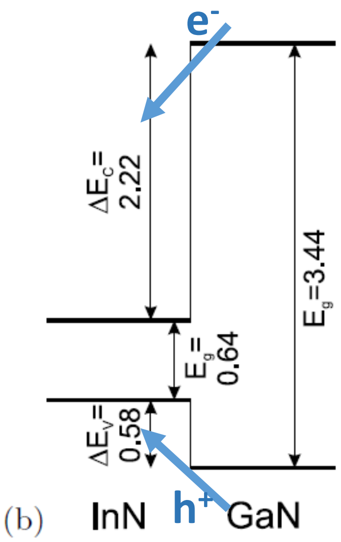




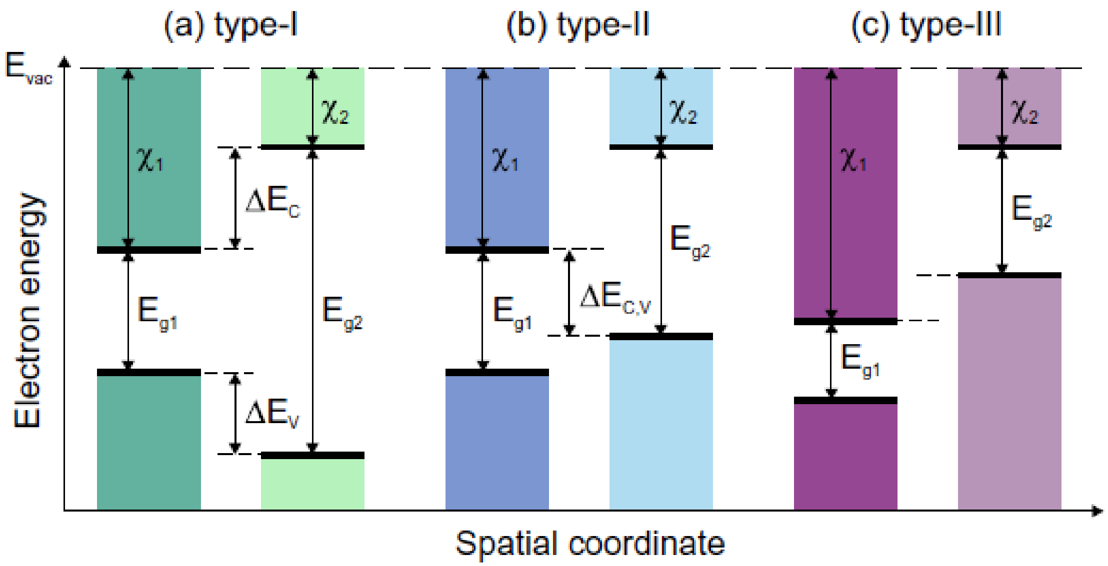
Grundmann Chapter 24

Impact of **confinement and heterostructures** on the carrier density





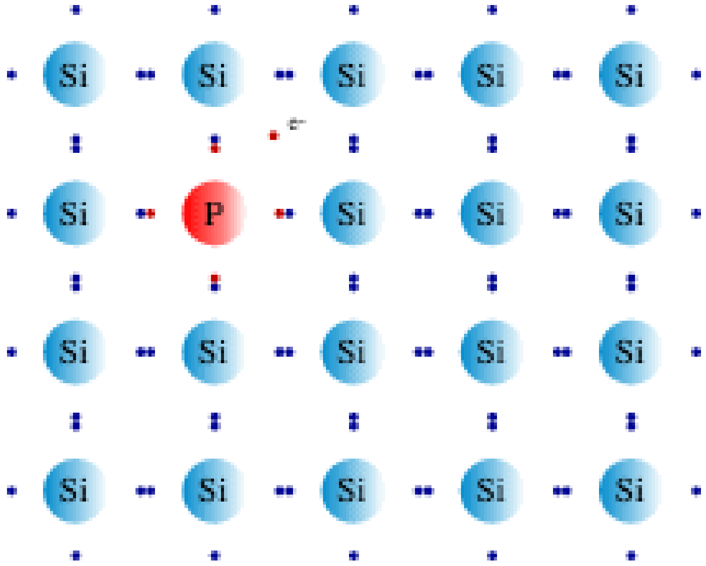
Topic of Classes 09-10

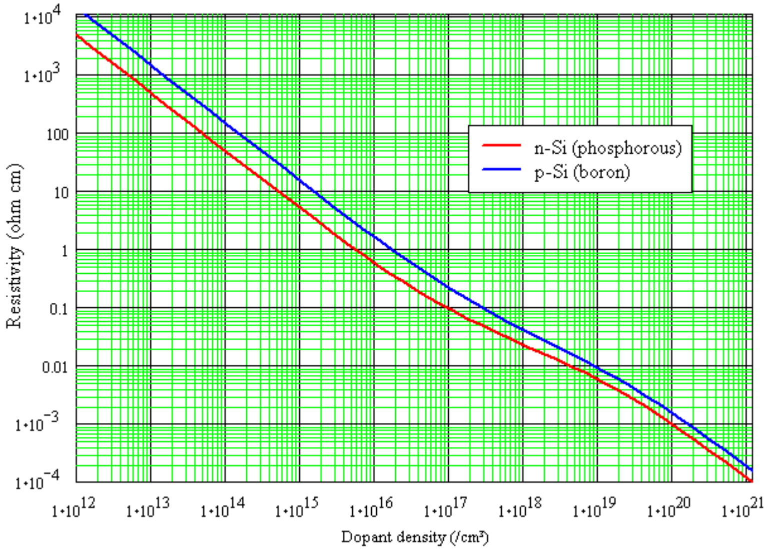


Grundmann 12.3

Impact of **impurities** on the carrier density

Topic of Class 05





INCREASE OF CONDUCTIVITY WITH IMPURITIES